UPPER PRIMARY STUDENTS’ ERRORS IN INTERPRETING

GRAPHICALLY-ORIENTED MATHEMATICS ASSESSMENT ITEMS

Often primary students are required to interpret or produce graphics (e.g., maps, number lines, graphs, tree diagrams) in mathematics assessment tasks. Thus, to become proficient in mathematics, students need to become graphicate as well as articulate, literate and numerate. This paper reports on one aspect of graphicacy, namely, students’ ability to interpret graphically-oriented mathematics items. Such items include graphics and text and might include symbols. Our study focussed on the errors that students make in the interpretation of graphically-oriented items because knowledge of errors is an important aspect of pedagogical content knowledge. Based on the literature on spatial intelligence and graphical understanding, we hypothesised that (1) Upper primary students would make perceptual and conceptualisation errors in interpreting graphics, and that (2) Upper primary students would make fewer perceptual errors than conceptualisation errors. These hypotheses were tested with a data set of final year primary students’ responses to 12 graphically-oriented interview items that were selected for theoretical diversity. Both hypotheses were supported. However, of particular note in our findings was that 66% of the students’ total errors related to the interpretation of a graphic with the remaining errors associated with the text (30%) or calculation (4%). The graphical errors were predominantly related to conceptualisation (56%) rather than perception (10%). Examples of perceptual and conceptualisation errors are presented in the paper. Based on our findings, we draw attention to three pedagogical issues. First, given the proportion of graphical errors on mathematics items, fostering graphical understanding should be an integral component of any mathematics program. Second, upper primary students may need support to develop their spatial perception skills although these skills are typically associated with the lower to middle primary years. Finally, interpreting graphically-oriented items is a complex process which requires students to interpret multiple representations including graphics simultaneously.

Key Words: Mathematics Education and Numeracy

INTRODUCTION

Over forty years ago, Balchin and Coleman (1965) identified graphicacy as one of four “aces” in an individual’s repertoire of essential skills for processing information — the other aces being literacy, oracy and numeracy. Although the importance of graphicacy has been recognised for many years in some curriculum areas, such as geography, (Boardman, 1976; Wilmot, 2002), graphicacy has been largely overlooked in other curriculum areas including mathematics. Typically, graphicacy is defined in one of two ways. Some view it narrowly as the ability to produce or interpret graphs (e.g., Gallimore, 1990) whereas others, us included, view it more broadly as the ability to produce or interpret all types of information presented in visual-spatial formats including graphs and tables, maps and number lines (e.g., Harris, 1996). In this paper, we focus on the interpretation of graphics in mathematics assessment items.

Graphics play an important role in communicating everyday mathematical information in our data-laden society. For example, many types of graphics are used to share information about mobile phones including a table of call charges for different phone plans and a map of the
coverage of particular carriers. Those who can interpret the mathematical information embedded in graphics are better able to make informed choices about mobile phones than those who cannot access the same information. Many other examples can be cited where graphics are commonplace in the communication of mathematical information. Thus, proficiency in interpreting graphics in mathematics is a divisor that empowers some individuals and disempowers others in everyday life. This divisiveness can also have a far reaching impact in schooling because graphics are used extensively in mathematics in instructional materials, texts, software and assessment. However, there is scant guidance for teachers about how to support students to interpret graphics and make meaning from them.

Understanding students’ errors is an important facet of pedagogical content knowledge in mathematics (Carpenter, Fennema, & Franke, 1996). Hence, this paper focuses on understanding the errors that students make interpreting graphics in mathematics because when teachers understand students’ errors they can provide strategic instruction to prevent the occurrence of errors or address them. To provide a background to students’ errors in interpreting graphics, we present an overview of spatial intelligence and graphical understanding. We then predict the types of graphical errors that students are likely to make and examine an existing data set to establish whether the data support our predictions. We conclude the paper with a discussion of pedagogical issues.

**BACKGROUND**

There is limited literature on students’ interpretation of graphics in mathematics. However, the literature on spatial intelligence, the types of graphics, and the perception and conceptualisation of graphics provides some guidance about the types of errors that students are likely to make when they interpret graphics on mathematical items.

**Spatial Intelligence**

Spatial intelligence is the ability to represent and interpret information presented in a visual-spatial format (Lohman, Pellegrino, Alderton, & Regian, 1987) and to reason with these representations (Rogers, 1995). These visual-spatial representations are diverse and include diagrams, drawings and mental images (Gardner, 1983). Based on Gardner’s developmental trajectory of intelligence, spatial intelligence has four stages which may overlap.

Stage 1 is the raw (patterning) ability associated with a particular intelligence, for example, a young child’s interest and competence in spatial activities such as drawing, puzzle completion and construction. Typically, early childhood educational experiences provide many opportunities for the development of this level of spatial ability.

Stage 2 involves symbol systems in particular domains. For example, drawing is important in mathematics (Tufte, 1983) and other subjects but drawing in mathematics is functional rather than artistic. The competent use of symbol systems in spatial activities relies on the following seven spatial perception skills (Del Grande, 1990):

1. Eye-motor co-ordination
2. Figure-ground perception
3. Perceptual constancy (The invariance of an object’s properties even though the object may look different when viewed from another perspective.)
4. Position-in-space perception (The relationship between two objects or an object and an observer such as the orientation of letters.)
5. Perception of spatial relationships (e.g., flips, slides, turns)
6. Visual discrimination and

In addition to creating graphics, students need to be able to interpret graphics prepared by others. According to Winn (1994), the comprehension of a graphic involves the interaction between a visual symbol system and perceptual and cognitive (i.e., conceptualisation) processes. The symbol system is composed of (1) visual elements (e.g., shapes) that represent objects or ideas and (2) the spatial relationships among the elements within the graphic (e.g., one shape inside another). Typically, students in the early to middle primary years encounter a range of graphics in mathematics that require them to use perceptual skills. For example, identifying the tallest child from a group of children of different heights in a drawing involves visual discrimination.

Stage 3 is the use of a notational system applicable within the domain. As this system is more abstract than in Stage 2, the producer of a graphic and its interpreter need to have shared meaning of conventions that might be used. For example, often a sloped line is used to indicate depth on a three-dimensional shape. Due to the widespread use of graphics in mathematics, students need to become adept at interpreting a variety of commonly used graphics. For example, knowledge of various diagrams is important for proficiency in mathematics (Eisenberg & Dryfus, 1986). Hence, during Stage 3, students need to develop their understanding of various aspects of graphics. According to Blackwell and Engelhardt (2002), these aspects are: (1) Signs (i.e., graphic elements, conventions, level of pictorial abstraction); (2) Graphic structures (e.g., a tree diagram, a linear diagram); (3) Meaning (i.e., correspondence between a representation and its meaning and classifications of information); and (4) Context-related aspects (i.e., the interaction between the person and the graphic, the cognitive processes involved in interpreting a diagram, and the cultural context of the graphic. Thus, this stage involves conceptualisation of the various aspects of the graphic.

Unlike Stages 1 and 2 in spatial intelligence, where development typically occurs through a variety of informal experiences, the use of a notational system is typically mastered through formal education (Gardner, 1993).

Stage 4 is the expression of a particular intelligence through vocational and avocational pursuits in adolescence and adulthood. For example, students with high spatial intelligence may pursue a career in architecture or a hobby such as pottery. A fuller discussion of the development of spatial ability is presented elsewhere (Author, 2000).

As with becoming literate and numerate, becoming graphicate in the primary years involves support for Stages 2 and 3 in the developmental trajectory of spatial intelligence. At Stage 2, becoming graphicate involves, for example, knowledge of the roles of various visual representations such as diagrams and sketches and the difference between them. Many students have difficulty with diagrams because they focus on the surface details rather than the structure of the information (Dufour-Janvier, Bednarz, & Belanger, 1987). Additionally, at this stage students need to master the spatial perception skills (e.g., visual discrimination) required to interpret graphics. At Stage 3, becoming graphicate involves knowledge of different types of graphics such as number lines and bar graphs. A discussion of the various types of graphics follows shortly. Thus, Stage 3 involves more sophisticated graphical knowledge than Stage 2 which is consistent with the notion that mathematical knowledge is hierarchical (Hinds, Patterson, & Pfeffer, 2001).
Understanding Graphics

In mathematics, Mackinlay (1999) argues that graphics can be categorised into six types of graphical “languages”, which represent mathematical relationships among perceptual elements and use particular encoding techniques. We use Mackinlay’s term “language” purposively because graphics have a communicative intent and also have their own unique elements and structures. These graphical languages are Axis, Opposed-position, Maps, Retinal-list, Connection and Miscellaneous languages. The languages are variously represented by a set of perceptual elements. These elements are position, length, angle, slope, area, volume, density, colour saturation, colour hue, texture, connection, containment, and shape (Cleveland & McGill, 1984). However, each element should not be interpreted in isolation. For example, whether a sloped line represents the actual slope of a hill, trend data on a graph or depth on a three-dimensional drawing of a cube requires attention to the other cues in the graphic and knowledge of graphical conventions. All graphical languages with the exception of the Miscellaneous language have a unique graphical structure. Miscellaneous language items can have various graphical structures (e.g., a calendar, a pie chart). See Table 1 for an overview of the six graphical languages and Figures 3A and 3B and the Appendix for examples of these items.

<table>
<thead>
<tr>
<th>Graphical Languages</th>
<th>Examples</th>
<th>Encoding Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axis Languages</td>
<td>Horizontal and vertical axes</td>
<td>A single-position encodes information by the placement of a mark on an axis.</td>
</tr>
<tr>
<td></td>
<td>Opposed-position Languages Line chart, bar chart, plot chart</td>
<td>Information is encoded by a marked set that is positioned between two axes.</td>
</tr>
<tr>
<td>Retinal-list Languages</td>
<td>Graphics featuring colour, shape, size, saturation, texture, orientation</td>
<td>Retinal properties are used to encode information. These marks are not dependent on position.</td>
</tr>
<tr>
<td>Map Languages</td>
<td>Road map, topographic map</td>
<td>Information is encoded through the spatial location of the marks.</td>
</tr>
<tr>
<td>Connection Languages</td>
<td>Tree, acyclic graph, network</td>
<td>Information is encoded by a set of node objects with a set of link objects.</td>
</tr>
<tr>
<td>Miscellaneous Languages</td>
<td>Pie chart, Venn diagram</td>
<td>Information is encoded with additional graphical techniques (e.g., angle, containment).</td>
</tr>
</tbody>
</table>

An individual’s ability to make meaning from a graphic relies on their perception and conceptualisation of the representation (Winn, 1994). For example, on the following number line (an Axis graphic) in Figure 1, students need to perceive the length of the number line and the position of the indicator, and conceptualise that this is a structured number line, and hence, the distance between two marks is a proportion of the overall length.
Our Predictions

From a review of the literature on spatial intelligence and graphics, we propose two hypotheses.

- **Hypothesis 1**: Upper primary students will make perceptual and conceptualisation errors in interpreting graphics.
- **Hypothesis 2**: Upper primary students will make fewer perceptual errors than conceptualisation errors.

DESIGN AND METHODS

To test our hypotheses, we undertook a confirmatory study (See Onwuegbuzie & Leech, 2005 for a discussion of the role of confirmatory studies). In this study, we assessed the replicability of hypotheses that emerged from the literature with the outcomes from an existing data set. The existing data set comprised responses from a class of 15 12-13-year-old Australian students on 12 interview items. These 12 interview items were the two hardest pairs of items in each graphical language from the 36 item Graphical Languages in Mathematics [GLIM] (See Author, 2007 for a fuller discussion of the GLIM test). In brief, this test consists of six sets of items of varying difficulty from each of the six graphical languages. Most items were drawn from published mathematics tests for students in the final three years of primary school. However, because insufficient mathematics items were available in particular languages, a few item content-free science test items were also included. One example of each of these pairs of items is show in Figure 3A or the Appendix.

During the individual interviews, students were presented with pairs of multiple choice items from the same graphical language. After selecting a response, the students were then asked to justify their selection. The interviewer probed students to fully explain their solution responses but provided no scaffolding. All interviews were videoed and transcribed to facilitate data analysis. The students’ multiple choice responses were scored 1 or 0 for correct or incorrect responses respectively. The data in this paper relates to students’ responses following the selection of an incorrect response. The data analysis process was undertaken with the students’ interview transcripts and the associated videotape available for clarification. The analysis process was a form of pattern matching (Yin, 1993) in which the data was tested against a theoretically-based pattern and the types of errors were identified.

RESULTS AND DISCUSSION

The results relating to each hypothesis are discussed in turn.
Hypothesis 1: Upper primary students will make perceptual and conceptualisation errors in interpreting graphics.

An analysis of students’ errors by themes and items is shown on Table 2. As anticipated, students made perceptual errors and conceptualisation errors interpreting the graphics. Examples of these types of errors are discussed shortly. Students also made calculation errors and text related (misreading or misinterpreting text) errors which are not discussed further. However, what was unanticipated was the high proportion of graphical errors. These errors accounted for 66% of the total errors which indicates that students’ knowledge of graphics is likely to compromise their mathematics performance substantially. A thematic analysis of the graphical errors made by students follows.

Figure 2. Proportion of error types.
Table 2. Errors by Item and Type

<table>
<thead>
<tr>
<th>Type of Graphic</th>
<th>Items</th>
<th>Perception Errors</th>
<th>Conceptualisation Errors</th>
<th>Calculation Errors</th>
<th>Text Errors</th>
<th>Total Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Axis</td>
<td>Butterfly</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2nd Axis</td>
<td>City to City</td>
<td>0</td>
<td>2*</td>
<td>1*</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>3 Opposed Position</td>
<td>Whale</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>4th Opposed Position</td>
<td>Meg’s Line Graph</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>5th Retinal List</td>
<td>Squares</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>6th Retinal List</td>
<td>Puzzle</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7th Map</td>
<td>Street Map</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8th Map</td>
<td>Bedroom Map</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9th Connection</td>
<td>Food Web</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>10 Connection</td>
<td>Board Game</td>
<td>0</td>
<td>9</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>11 Miscellaneous</td>
<td>Paper Game</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12th Miscellaneous</td>
<td>Pie chart shape budget</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Total Errors</td>
<td>5</td>
<td>28</td>
<td>2</td>
<td>15</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

*This item is shown in Figure 3A or the Appendix. * One student made more than one type of error per item.

**Theme 1: Perception Errors**

Students made five spatial perception errors in total. These errors were made on only two of the 12 items (Table 2). Four of these errors occurred on the Squares item (Retinal list) (Appendix, Item 5) and the other on the Bedroom Map (Map) which showed a bird’s eye view. All students who had difficulty with the Squares item attempted to mentally rotate, flip, and fit the shapes together to make the shape depicted. However, despite employing appropriate processes, the task was apparently too challenging for them. Eliza (E), for example, commented to the interviewer (I) that attempting to solve the item made her brain hurt, indicating that this task was difficult for her.

E: I didn’t really get that so I sort of just tried to fit the map using that shape flipped, turned and rotated and all that stuff and I just sort of ended up going with A.
I: Is there any reason why you went with A?
E: No just my brain hurts too much (emphasis added).
I: Okay so you just guessed basically?
E: Yeah
I: Okay

However, it should not be assumed that those who had difficulty with some spatial perception processes on a particular type of graphical language would have difficulty with other types.
Two of the four students (Eliza, Cam) who had difficulty with the spatial perception component of the Squares item (Retinal list) made few (Eliza) or no errors (Cam) on other graphical language items.

**Theme 2: Conceptualisation Errors**

More than half of the total errors (56%) related to students’ conceptualisation of the graphic (Table 2). These errors were of two types.

First, there were errors that indicated a lack of understanding of a convention. For example, although Harriet knew that the key was important on the Food Web item (Connection), she was unable to comprehend the meaning of the arrows in the key (Appendix, Item 9).

![Student explanation](emphasis added)

This science-oriented item is particularly complex because in addition to reasoning about the graphic (i.e., placement of boxes, arrows connecting boxes, directionality of those arrows), students also need to reason about the text (i.e., the text of the question, text associated with the key, text in each box) and make deductions from the information provided in both graphical and textual formats.

Second, there were errors that related to an interaction between the use of a convention and perception. For example, on the Line Graph item (Opposed position) (Figure 3A), Helen (H) identified the dots on the graph as rests rather than focussing on the labelled parts of the graph to reach a solution. As the X axis represented “time of day” and the Y axis represented “distance in km”, rests were represented by horizontal lines. Helen’s explanation for her selection of the incorrect response follows. The other five students who were incorrect on this item made similar errors.

H: Four hours … well I … first of all I found where the animal plankton was and then I looked at it and I figured out that if the jewfish died than the shark wouldn’t die because it’s the (unclear) but anyway and then I looked at the key and I never really kind of understand these things because I don’t know it’s where the arrow is, is eaten by that or where the end of the arrow is eaten by that but so then I looked down here and I think animal plankton is pretty small so just up above that and I figured out the shark is at the top so the stuff at the bottom must be eaten by the shark the top of it so I saw that plant plankton … oh then I got saw that plant plankton eats animal plankton yeah so if the animal plankton died, the plant plankton would have nothing to eat yeah I think so, I don’t really understand the arrows. (emphasis added)

I: And four hours, where does your four hours come from?
H: It passes through four boxes.
Helen’s difficulty related to a lack of understanding of the conventions of the graphic and an over reliance on the perceptual elements (i.e., the dots). The inclusion of the dots on this graphic was presumably to draw students’ attention to a change in the slope of the line. However, instead of supporting the interpretation of the graphic, the dots acted as a perceptual distractor. Other members of our project team have trialled this item both with and without the dots (Figures 3A and 3B) with a different set of students and found that students were more successful on this item without the dots (Logan & Greenlees, forthcoming). Logan and Greenlees reported, for example, clear differences in one student’s reasoning with and without the dots (See Table 3). Students’ responses to the dots on this item is an example of how perceptual cuing can inhibit understanding. For some students, the inclusion of dots was an overly strong perceptual cue that diverted rather than supported their attention (See Alexander, Kulikowich, & Schulze, 1994, for a discussion of the negative effects of seductive detail).

Table 3. A Student’s Reasoning With and Without a Perpetual Cue

<table>
<thead>
<tr>
<th>With the dots</th>
<th>“I chose 1 hour because she started at 6 am and she stopped at 7 am because here it has a dot where it was a new hour” (emphasis added)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without the dots</td>
<td>“I chose two hours because on the graph it keeps on going up until she gets from 10 am to 12 pm and then it just goes straight so she’s not moving any distance which means she must have stopped”</td>
</tr>
</tbody>
</table>

(Queensland Studies Authority, 2002a) (Logan & Greenlees, forthcoming)

Figure 3A. Opposed position item (Item 4) with a perceptual cue.  
Figure 3B. Opposed position item without a perceptual cue.

Hypothesis 2: Upper primary students will make fewer perceptual errors than conceptualisation errors.

The data support the hypothesis that upper primary students will make fewer perceptual errors than conceptualisation errors because students made 5 and 28 errors respectively (Table 2). However, this hypothesis needs to be tested further because clearly the type of item impacted on the type of error. Perceptual errors were made only on Item 5 (Retinal list) and Item 8 (Map). In contrast, conceptualisation errors were made on five different items: Items 1 and 2 (Axis), Item 4 (Opposed position) and Items 9 and 10 (Connection). Thus, in total graphical errors were made on seven of the 12 items. In each case, there were either perceptual errors or conceptualisation errors but not both which suggests that items from particular graphical languages may evoke distinctive types of processing, and consequently, errors.
CONCLUDING COMMENTS

Our study revealed three points of pedagogical interest. First, because a large proportion of students’ errors related to graphicacy rather than literacy or numeracy, there is a need to review, and if necessary revise, mathematical programs to ensure that all students are provided with opportunities to become graphicate. Consistent with the recommendations of the Rand report — *Mathematical Proficiency for All Students* (Ball, 2004) — such a program should incorporate explicit attention to the practices of successful mathematical users. Elsewhere we have reported that different approaches are adopted by successful and unsuccessful students on graphically-oriented mathematics items (Author, 2007).

Second, this study revealed that some upper primary students have limited mastery of spatial perception skills. Lack of mastery of foundational knowledge is problematic because it inhibits the development of more sophisticated knowledge (Hinds et al., 2001) Hence, limited mastery of spatial perception skills constitutes a serious shortcoming in these students’ mathematical knowledge and they need urgent and effective support. However, as with many educational programs there is the likelihood that some students will experience serious difficulties. Thus, just as there are sufferers of dyslexia and dyscalculia who experience difficulties with words and numbers respectively, it is plausible that some individuals may also suffer from a difficulty with graphics — dysgraphica1.

Third, the study highlights the complexity of interpreting mathematical items that incorporate graphics as well as text and/or symbols. Similar to using multiple foreign languages to communicate parts of the information about a topic, some mathematical items use multiple representations simultaneously to convey complementary mathematical information about a situation. Thus, the interpretation of a mathematics item incorporating graphics, text and/or symbols is akin to functioning in a “multi-lingual” environment. Not only do students need to interpret and reason about graphics differently to text and symbols but they need to vary their reasoning according to the particular type of graphical language item.

In summation, it is timely for graphicacy to be an educational goal in mathematics for all students. To do otherwise casts mathematics in the unsatisfactory role of gatekeeper (Stinson, 2004): “Mathematics should not be used as an instrument for stratification but rather an instrument for empowerment!” (p. 16).

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REFERENCES


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1 Dysgraphica is also sometimes used to describe a condition in which someone experiences serious difficulties with writing.


APPENDIX*

Axis: Item 2 - City to City

Bay City                            Exton                     Yardville

On the road shown above, the distance from Bay City to Exton is 60 kilometres. What is the distance from Bay City to Yardville?

(National Centre for Educational Statistics, 2003, p. 19)

Map: Item 7 - Street Map

Bill leaves the pool. He drives north and takes the first road on the right, then the second road on the left. Which road is he in?

(Queensland Studies Authority, 2002a, p. 7)

Retinal List: Item 5 - Squares

This shape was used to make different designs.

Which one of the following designs cannot be made using only four of the shapes above?

(Educational Testing Centre, 2001)

Connection: Item 9 - Food Web

The animals in this food web eat only what is shown.

If all the animal plankton die which of the following will also die?

(Educational Testing Centre, 1995, p. 4.)

Miscellaneous: Item 12 – Pie Chart

In 2004, Jemma budgeted $30 on clothes. Approximately how much money did she get that year?

(Queensland Studies Authority, 2002b, p. 6)
* Multiple choice response options were also presented with these items to the students.